Essay

$$$001
The current state of the use of the technology of collective mutual teaching in secondary school
$$$002
The teacher's function in the collective mutual teaching of mathematics in secondary school
$$$003
The state of implementation of the principle of continuity in the content of secondary school mathematics (algebra, geometry, mathematical analysis)
$$$004
The importance of empirical methods (control, survey, conversation) in teaching school mathematics
$$$005
Bloom's taxonomy of evaluation of expected results in school teaching mathematics
$$$006
The role of algorithmic technology in the development of mathematical thinking of pupils
$$$007
Simpson's universal formula as a means of integrated training in planimetry and stereometry courses.
$$$008
The situation of traditional teaching of school mathematics
$$$009
The function of a teacher in traditional and innovative teaching of school mathematics
$$$010
Reasons for the transition to innovative teaching of school mathematics
$$$011
Teaching school mathematics using the technology of consolidation of didactic units
$$$012
Numerical series and its properties
$$$013
The connection between the subjects of algebra, geometry and mathematical analysis at school.
$$$014
Concepts of knowledge, skills and abilities in teaching school mathematics.
$$$015
The role of text tasks in the cognitive process
$$$016
The value of construction tasks in teaching a school geometry course.
$$$017
Elements of mathematical statistics in secondary school

$$$018
Types of profile teaching of school mathematics.
$$$019
The concepts of numbers, measures and quantities in school mathematics.
$$$020
Concepts of operator, functional and function in school mathematics.

Questions

001
Fundamentals of the theory of divisibility. The divisibility relation and its properties. Properties of prime and composite numbers. Euclid 's algorithm.
002
Basic concepts and theorems about matrices and determinants. Methods of finding the minor, algebraic complement and inverse matrix.
003
Basic concepts and theorems related to the system of linear equations and methods of their solution.
004
The ring of complex numbers. The relationship between the writing of complex numbers in various forms and their geometric meanings.
005
Vector space, basic concepts and theorems of vector algebra. Linear dependence of vectors.
006
Polynomials with one variable. Divisibility of polynomials. Euclid's algorithm. Ways to find the roots of a polynomial.
007
Linear operators and their matrix. The relationship between the matrix and the transformation.
008
Types of linear transformations. The image and core of the transformation.
009
The eigenvalue and eigenvector of the linear operator.
010
The main elements of the theory of comparisons . Euler and Fermat theorems.
011
Vectors. Linear operations on vectors. Scalar and vector product of two vectors and their properties.
012
Different ways to set a straight line on a plane. The relative position of two and three straight lines. The angle between two straight lines.
013
Ellipse. Investigation of an ellipse by its canonical equation. Foci, eccentricity, and directrixes of the ellipse.
014
Hyperbola. Investigation of a hyperbola by its canonical equation. Foci, eccentricity, directrixes, and asymptotes of the hyperbola.
015
Different ways to define a plane in space. The relative position of two and three planes. The distance from the point to the plane.

016
Different equations of a straight line in space.The distance from a point to a straight line in space.
017
The relative position of a straight line and a plane in space. The angle between a straight line and a plane.
018
The concept of a surface. Surfaces of rotation (spherical, cylindrical, and conical surfaces) and their properties.
019
Canonical equations of second-order surfaces and their types. Rectilinear generators of second-order surfaces.
020
Overview of the general theory of second-order surfaces. Classification of second-order surfaces.
021
Bolzano - Weierstrass theorem and Cauchy criterion for numerical sequences.
022
Continuity of a function of one variable, points of discontinuity and their classification. Properties of functions that are continuous on a segment.
023
Rolle's, Lagrange's, Cauchy's theorems.
024
Taylor's formula for a function of one variable. Expansion of functions in power series. Expansion of functions e^x, ln (1+x).
025
Extremum of a function of several variables. Necessary and sufficient conditions for the extremum of a function of several variables.
026
The definite integral as the limit of integral sums, its properties and connection with the indefinite integral. Change of a variable in a definite integral.
027
Improper integrals of the first and second kind.
028
Higher-order differential equations. General concepts. Equations that allow lowering the order.
029
Linear differential equations of the nth order.
030
Linear homogeneous equations.
031
Linear homogeneous equations with constant coefficients and Euler equations.
032
Linear inhomogeneous equations.
033
Linear inhomogeneous equations with constant coefficients.
034
Integration of differential equations using series.
035
The concept of a boundary value problem.
036

A curvilinear integral of the first kind. Ways of calculating curvilinear integrals of the I-kind.
037
A curvilinear integral of the second kind. Ways of calculating curvilinear integrals of the II-kind.
038
The relationship between a curvilinear integral of the first kind and a curvilinear integral of the second kind.
039
Problems leading to the concept of a double integral. The concept of a double integral. The concept of a repeated integral.
040
Substitution of variables in the double integral.
041
Algebra of probability theory. Determination of probabilities.
042
Local and integral Moivre-Laplace theorems.
043
Random variables and their numerical characteristics.
044
The law of large numbers. The Chebyshev and Bernoulli theorems.
045
Elements of mathematical statistics.
046
Propositions. Boolean algebra of propositions.
047
Functions of the algebra of logic and ways to set them. Truth tables.
048
Decompositions of Boolean functions over variables. Perfect disjunctive and conjunctive normal forms.
049
Closed classes and completeness of systems of functions of the algebra of logic. Post's theorem.
050
Minimization problems for logical functions. Closed disjunctive normal forms.
001
Investigate the linear dependence of the system of vectors: $\cos\mathbf{x},\quad \sin\mathbf{x},\quad \sin 2\mathbf{x}$ on the interval $(-\pi/2,\ \pi/2)$.
002
Find the coordinates of the vector $\mathbf{x}$ in the basis $(\mathbf{e}_1',\quad \mathbf{e}_2',\quad \mathbf{e}_3')$, if it is given in the basis $(\mathbf{e}_1,\quad \mathbf{e}_2,\quad \mathbf{e}_3)$:
$$\begin{cases} \mathbf{e}_1' = \mathbf{e}_1 + \mathbf{e}_2 + 2\mathbf{e}_3, \\ \mathbf{e}_2' = 2\mathbf{e}_1 - \mathbf{e}_2, \\ \mathbf{e}_3' = -\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3, \end{cases}$$
$$\mathbf{x} = \{6,\quad -1,\quad 3\}.$$
003
The matrix of the linear operator f in the basis e_1, e_2, e_3 of some linear space is given:
$$A = \begin{pmatrix} 0 & 1 & 0 \\ 6 & 3 & 2 \\ 3 & 0 & 1 \end{pmatrix}.$$

Is the vector x an eigenvector of this linear operator? If it is an eigenvector, then what eigenvalue does it belong to?
1) $x = -e_1 + 2e_2 - 2e_3$,
2) $x = e_1 - 3e_3$.
004
Using the Gram-Schmidt orthogonalization process, construct an orthonormal basis $(f_1, f_2, \dots, f_m)$ on a linear shell $L = L(g_1, g_2, \dots, g_m)$:
$g_1 = (1; -2; 5)$, $g_2 = (3; -1; 5)$, $g_3 = (5; -5; 3)$.

005
Two linear operators are given. Using matrix calculations, find operators expressing variables x_1'', x_2'', x_3'' through x_1, x_2, x_3. Justify your answer.

$$\begin{cases} x_1' = x_1 - 3x_2 - 2x_3 \\ x_2' = -4x_1 + x_2 + 2x_3 \\ x_3' = 3x_1 - 4x_2 + 5x_3 \end{cases} \qquad \begin{cases} x_1'' = -x_1 - 2x_2 + x_3 \\ x_2'' = 3x_1 + x_2 - x_3 \\ x_3'' = x_1 - x_2 + x_3 \end{cases}$$

006
Given:
$x = \{x_1, \quad x_2, \quad x_3\}$, $Ax = \{x_2 - x_3, \quad x_1, \quad x_1 + x_3\}$, $Bx = \{x_2, \quad 2x_3, \quad x_1\}$. Find: ABx. Justify the answer.

007

To investigate the system of vectors for linear dependence:
$1 + \mathbf{x} + \mathbf{x}^2, \ 1 + 2\mathbf{x} + \mathbf{x}^2, \ 1 + 3\mathbf{x} + \mathbf{x}^2$, $(-\infty, \ +\infty)$.
008
Prove the signs of the divisibility of a number by 2, 3, 4, 5, 11 based on the divisibility of numbers defined by Pascal.
009
Find the eigenvalues and eigenvectors of the matrix. $\begin{pmatrix} 6 & -2 & -1 \\ -1 & 5 & -1 \\ 1 & -2 & 4 \end{pmatrix}$.

010

Give examples of the generalized Vieta theorem and its application in solving equations.

011
Find the coordinates of the vector $\mathbf{X}$ in the basis $(\mathbf{e}_1', \quad \mathbf{e}_2', \quad \mathbf{e}_3')$, if it is given in the basis $(\mathbf{e}_1, \quad \mathbf{e}_2, \quad \mathbf{e}_3)$: $\begin{cases} \mathbf{e}_1' = \mathbf{e}_1 + \mathbf{e}_2 + 3\mathbf{e}_3, \\ \mathbf{e}_2' = (3/2)\mathbf{e}_1 - \mathbf{e}_2, \\ \mathbf{e}_3' = -\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3, \end{cases}$
$\mathbf{x} = \{1, \quad 2, \quad 4\}$.

012

In the ABCD parallelogram, the following is given: $\overline{AE} = \frac{1}{2}\overline{AE} = \overline{a}, \overline{AF} = \frac{1}{2}\overline{AD} = \overline{b}$,
find vectors: $\overline{CB}, \overline{CD}, \overline{AC}, \overline{DB}$.

013

Triangle ABC is defined by the coordinates of its vertices: A(3,2), B (-2,0), C(-1,1). To find: 1) the cosine of the angle ∠A ; 2) the area of the triangle ΔABC .

014

Given: $\overline{a} = \overline{3i} + \overline{6j}, \quad \overline{b} = \overline{2i} - \overline{9j}$. Find: $|\overline{a}|, |\overline{b}|, (\overline{a}, \overline{b})$. Explain their geometric meanings.

015

Write the equation of the straight line passing through the point M (3,2) collinear to the vector $\vec{p} = (2,1)$. Justify the answer.

016

Write the equations of the straight lines passing through the point A (5, -1) and parallel / perpendicular / to the straight line y = 2x-3. Justify the answer.

017

Find the distance between foci and eccentricity of an ellipse $25x^2 + 9y^2 = 225$. Explain their geometric meanings.

018

Find the angle between two straight lines: $l_1: 3x - 4y = 5, \quad l_2: y = 5x - 3$.

Justify the answer.

019

Find the distance from the point M (5, -2) to the line $l: 3x + 4y - 5 = 0$. Justify the answer.

020

Find the distance from the point M (2, -3,0) to the plane $2x - 2y + z - 5 = 0$.
Justify the answer.

021

For a given series:1) find the sum (S_n) of the first n terms of the series; 2) using the definition of convergence, prove the convergence of the series;

3) find the sum of the series: $\sum_{n=1}^{\infty} \frac{1}{(2n+7)(2n+9)}$.

022

Investigate series for convergence:

а) $\sum_{n=1}^{\infty} \left(\frac{2n+1}{41n^2+1}\right)^2$, б) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)ln(2n+1)}$.

023

Investigate the alternating series for convergence and for absolute convergence:
$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(n+1)\cdot 3^n}$.

024

Find the convergence region of the series: $\sum_{n=1}^{\infty}\frac{nx^n}{2^{n-1}.3^n}$.

025

Find the convergence region of the series: $\sum_{n=1}^{\infty}\frac{2^n x^n}{2n-1}$.

026

Expand the periodic function f (x) (ω=2π period) in a Fourier series on a given interval [-π;π] , if $f(x)=\begin{cases}-x-1/2, & -\pi \le x < 0,\\ 0, & 0 \le x \le \pi.\end{cases}$

027

Find the domain: $z=\frac{3x+y}{2-x+y}$.

028

Find the partial derivatives and differentials of the following function: $z=ln(y^2-e^{-x})$.

029
Calculate the curvilinear integral:
$\int_{L_{OA}}(x^2+y^2)dx+2xydy$, here L_{OA} - the arc of a cubic parabola $y=x^3$ from point O(0,0) to point A(1,1) .

030

Calculate the double integral $\iint_D (x+y)dxdy$, over the region D bounded by the following lines: D: y^2=x, $y=x$.
031

Find a solution to the Cauchy problem : $y''-4y'+5y=0, y(0)=0, y'(0)=1$

032

Find a solution to the Cauchy problem : $y''+y=4e^x, y(0)=4, y'(0)=-3$

033

Solve the differential equation: $y'' - y = e^x$

034

Calculate the limit of a sequence: $\lim_{n\to\infty}(\sqrt{2}\cdot\sqrt[4]{2}\cdot\sqrt[8]{2}....\sqrt[2^n]{2})$.

035

Find the integral: $\int e^x \cos x dx$.

036

Find y'_x, if the function is given parametrically: $\begin{cases} x = a\cos^2 t \\ y = b\sin^2 t \end{cases}$.

037

Find the extremum point $P(x_0, y_0)$ of a function of two variables $z = (x-2)^2 + 2y^2 - 10$. Justify the answer.

038

Applying the Lagrange theorem, find the point C of the function $y = \sqrt{x}$ on the segment $[0;1]$.

039

Investigate the following integrals for convergence: $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{ctgx}}$.

040

Investigate the following integrals for convergence: $\int_0^{\infty} \frac{\sin axdx}{b^2 + x^2}$.

041

A list of possible values of a discrete random variable X is given: x1 = -1, x2 = -2, x3 = -3. And also the mathematical expectations of this value and its square are known: M (X) = 2.3; M (X2) = 5.9. Find the probabilities corresponding to the possible values of X.

042

Find the dispersion and the mean square deviation of a discrete random variable X given by the distribution law:

X	-5	2	3	4
P	0.4	0.3	0.1	0.2

043

A continuous random variable X is given by the differential function $f(x) = 2/3 \sin 3x$ in the interval $(0, \pi/3)$; outside this interval $f(x) = 0$. Find the probability that X takes a value belonging to the interval $(\pi/6, \pi/4)$.

044

A continuous random variable X is given by the differential function $f(x) = 2x$ in the interval (0,1); outside this interval $f(x) = 0$. Find the mathematical expectation and dispersion of X.

045

Find an empirical function for a given sample distribution:

x_i	1	4	6
n_i	10	15	25

and draw a graph of this function.

046

Investigate the completeness of the system of functions –

$D=\{x_1+ x_2 \to \neg x_3 , \neg(x_1 \vee x_2 +x_3), \neg(x_2 x_3), x_1 / 1 \}$.

047
Convert the formula to disjunctive normal form and reduce it to abbreviated form:
$F(x_1,x_2,x_3,x_4) = (x_1 \sim \neg x_3)(\neg x_2 \sim x_4) \vee (x_1/\neg x_4) \vee x_1 \neg x_2 x_3 \neg x_4$

048
Construct a shortened electronic contact diagram of a given function:
$F(x_1,x_2,x_3) = (x_1x_2 \oplus x_2x_3) \vee (\neg x_1 \neg x_2 \to x_3) \vee x_1x_2 \vee x_3$.

049

Write perfect disjunctive and conjunctive normal forms for a given function:
$f(x_1,x_2,x_3) = x_1x_2x_3 / x_2 \neg x_3 / \neg x_1x_2 \to \neg x_1 \neg x_2 \neg x_3$.

050
Reduce this formula to closed disjunctive normal form:

$\eta = \neg x_1\neg x_2\neg x_3 \vee x_3\neg x_1\neg x_2 \vee x_2 \neg x_1 x_3 \vee x_1 x_2 x_3 \vee \neg x_3 x_1 x_2 \vee x_1 \neg x_2 \neg x_3$.

001
Computer applications for use in teaching mathematics at school and their effectiveness.
002

Actual problems of teaching mathematics in the context of digitalization of education.
003
Numerical systems, methods of its construction and the place of numerical systems in mathematics.
004
Methods of studying ordinary fractions at school.
005
Methods of studying negative numbers at school.
006
Methods of introducing and teaching irrational numbers at school.
007
Methods of teaching pupils approximate calculations.
008
Ways of introducing the concept of identity in school mathematics.
009
Methods and techniques of teaching pupils to solve equations.
Direct and inverse operations on numbers.
010
The main ways of solving text problems in the school mathematics course.
011
Consideration of methods for solving linear equations and systems of equations in school mathematics.
012
Methods of teaching signs of divisibility of natural numbers in school mathematics.
013
Methods of teaching students the concept of rational numbers and their properties.
014
Methods of teaching direct and inverse operations on numbers.
015
Teaching pupils to solve problems for composing equations and their stages.
016
Teaching pupils to distinguish a complete square from a square trinomial.
017
Teaching pupils to purposefully perform identical transformations.
018
Methods of studying the course of planimetry.
019
Geometric methods for solving problems in geometry.
020
Algebraic methods for solving problems in geometry.
021
Combined methods for solving problems in geometry.
022
Methods of studying the course of stereometry.
023
An axiomatic method for solving construction problems in space.
024
A projective method for solving construction problems in space.
025
The method of the geometric location of points for solving construction problems in space.
026
The main stages of learning geometry.

027
The use of visual aids in the study of stereometry..
028
The concept of a series in mathematics. Find the sum of a series: $arctg\frac{1}{2} + arctg\frac{1}{8} + \cdots + arctg\frac{1}{2n^2} + \cdots$.
029
Convergence of series. Investigate the series for convergence: $\sum_{n=1}^{\infty} nsin\frac{2\pi}{3^n}$.
030
Convergence of series. Investigate a series for absolute or conditional convergence: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}\sqrt[5]{(n+1)^3}}$.
031
Convergence of series. Find the convergence region of the series: $\sum_{n=1}^{\infty} x^n tg\frac{x}{2^n}$.
032
Convergence of series. Find the radius of convergence of the series: $\sum_{n=1}^{\infty} \frac{n!x^n}{n^n}$.
033
Fourier series and its role in mathematical physics. Expand in a Fourier series the function f (x), given on the interval (0;π), continuing it in an even and odd way: $f(x) = sh2x$.
034
Partial derivatives of functions of many variables. Find the values of partial derivatives $\frac{\partial z}{\partial x}$ и $\frac{\partial z}{\partial y}$ at a given point $M_0(0,2,1)$, if: $3x^2y^2 + 2xyz^2 - 2x^3z + 4x^3y = 4$.
035
The concept of an integral in school mathematics.
Change the order of integration: $\int_0^1 dx \int_0^{x^{\frac{2}{3}}} f(x,y)dy + \int_1^2 dx \int_0^{1-\sqrt{4x-x^2-3}} f(x,y)dy$.
036
Using the second-order derivative, find the extremum of the function : $f(x) = x^4 - 4x^3 + 4x^2$
037
Applying Rolle's theorem, find the point C of the function $f(x) = x^3 - x^2 - x + 1$ on the segment [-1;1].
038
Applying Lagrange's theorem, find the point C of the function $f(x) = e^x$ on the segment [0; 1].
039
Calculate the following improper integrals: $\int_0^{\infty} \frac{xdx}{\sqrt{e^{2x} - 1}}$. Justify the answer.
040
Calculate the following improper integrals: $\int_0^{\infty} e^{-ax} \cos bxdx$. Justify the answer.
041
The vertices of the triangle ΔABC are given ΔABC: A(5,-3) , B(-2,-1), C(2,5). Write the equations of the lines containing the median of AD and the height of CH of this triangle.
042
Write the equations of the straight lines passing through the point M (-2.5) and parallel / perpendicular / straight line AB, if A (2, -3), B (5.1).
043

Find the distance from the point M (5, -3, 0) to the straight line $\begin{cases} 2x+y-z=0 \\ 5x-3y+z+2=0 \end{cases}$ . .
044
Find the distance from the point M (-2, 5, 1) to the plane passing through three points: A(-3, 0, 5), B(0, -2, 1), C (5, 2, 4).
045
Find the angle between the planes: P_1 : 2x-3y+z+1=0, P_2 : 5x+2y-2z+5=0.
046
The concept of symmetric polynomials. Express the given symmetric polynomials in terms of the basic symmetric polynomials:
$f(x) = x_1^3 + x_2^3 + x_3^3 - 2x_1^2x_2^2 - 2x_1^2x_3^2 - 2x_2^2x_3^2$
047
The dimension of a linear space. Find the basis and determine the dimension of the linear solution space of the system: $\begin{cases} 3x_1+x_2-8x_3+2x_4+x_5=0, \\ 2x_1-2x_2-3x_3-7x_4+2x_5=0, \\ x_1+11x_2-12x_3+34x_4-5x_5=0. \end{cases}$
048
The main elements of probability theory considered in school mathematics. Basic concepts of probability theory. A point is thrown at random inside a circle of radius R. Find the probability that the point: a) will be inside a square inscribed in a circle; b) will not be inside a regular triangle inscribed in a circle?
049
The main elements of probability theory considered in school mathematics. 20 textbooks are placed in random order on the library shelf, 5 of them are bound. The librarian takes 3 textbooks at random. Find the probability that at least one of the textbooks taken will be bound.
050
The main elements of probability theory considered in school mathematics. Basic concepts of probability theory. To destroy the enemy's shelter, it is enough to hit one aircraft bomb. Find the probability that the shelter will be destroyed if 4 bombs are dropped on it, the probability of hitting which are respectively equal to: 0,2; 0,5; 0,6 and 0,4.